Answers: Calculus Rectangular Complex Number Practice #3

1. Give the roots of $9x^2 - 12x + 22$ in exact values, using surds if required.

2. Give the roots of $kx^2 - 4kx + 12$, using surds if required.

3. What is the remainder if $2x^3 + 4x^2 + 5x + 3$ is divided by $x + 2$?

4. Show, using the Factor Theorem, that $x + 1$ is a factor of $9x^3 + 15x^2 + 35x + 29$.

5. For what values of k does $5x^2 + x + k$ have only complex roots?

6. $(\sqrt{2})i$ is a root of $3x^3 - x^2 + ax + b$. Find the other two roots.

7. Write $\frac{6 - ki}{6 + ki}$ in the form $a + bi$

8. Show that if $u = 3 + i$ then $\left| \frac{1}{u} - \frac{1}{\bar{u}} \right| = 0.2$

Answers: Calculus Rectangular Complex Number Practice #3

1. Give the roots of $9x^2 - 12x + 22$ in exact values, using surds if required.

$$\frac{--12 \pm \sqrt{12^2 - 4\times 9\times 22}}{2\times 9} \quad = \frac{2}{3} \pm \frac{\sqrt{-628}}{\sqrt{324}} \quad = \frac{2}{3} \pm \sqrt{2}\, i$$

2. Give the roots of $kx^2 - 4kx + 12$, using surds if required.

$$\frac{--4k \pm \sqrt{(4k)^2 - 4\times k\times 12}}{2\times k} \quad = \frac{4k}{2k} \pm \frac{\sqrt{16k^2 - 48k}}{\sqrt{4k^2}} \quad = 2 \pm \sqrt{\frac{4k-12}{k}} \quad = 2 \pm 2\sqrt{\frac{k-3}{k}}$$

3. What is the remainder if $2x^3 + 4x^2 + 5x + 3$ is divided by $x + 2$?

$f(5) = 2(^{-}2)^3 + 4(^{-}2)^2 + 5(^{-}2) + 3 = {}^{-}7$ so the remainder is $^{-}7$

4. Show, using the Factor Theorem, that $x + 1$ is a factor of $9x^3 + 15x^2 + 35x + 29$.

$f(^{-}1) = 9(^{-}1)^3 + 15(^{-}1)^2 + 35(^{-}1) + 29 = 0$ so $x + 1$ is a factor

5. For what values of k does $5x^2 + x + k$ have only complex roots?

complex roots if $\Delta = b^2 - 4ac < 0$

$1^2 - 4 \times 5 \times k < 0$, so $^{-}20k < {}^{-}1$ $\quad k > 0.05$ (note sign flip for $\div\ {}^{-}20$)

6. $(\sqrt{2})i$ is a root of $3x^3 - x^2 + ax + b$. Find the other two roots.

Factor theorem $3(\sqrt{2}i)^3 - (\sqrt{2}i)^2 + a(\sqrt{2}i) + b = 0$

Real parts need to equal zero: $-(\sqrt{2}i)^2 + b = {}^{-}2i^2 + b = 0 \quad \Rightarrow b = {}^{-}2$

Imaginary parts: $3(\sqrt{2}i)^3 + a(\sqrt{2}i) = 0$, so $3 \times 2i^2(\sqrt{2}i) + a(\sqrt{2}i) = 0 \quad \Rightarrow a = 6$

$3x^3 - x^2 + 6x - 2$ has roots (graphics calculator) of $\sqrt{2}i$, $^{-}\sqrt{2}i$ and $\frac{1}{3}$

7. Write $\frac{6-ki}{6+ki}$ in the form $a + bi$

$$\frac{(6-ki)(6-ki)}{(6+ki)(6-ki)} \quad = \frac{36 - 12ki + k^2 i^2}{36 + k^2} \quad = \frac{36 - k^2}{36 + k^2} + \frac{-12k}{36 + k^2}\, i$$

8. Show that if $u = 3 + i$ then $\left| \frac{1}{u} - \frac{1}{\bar{u}} \right| = 0.2$

$$\left| \frac{1}{u} - \frac{1}{\bar{u}} \right| \quad = \left| \frac{1}{3+i} - \frac{1}{3-i} \right| \quad = \left| \frac{3-i}{(3+i)(3-i)} - \frac{3+i}{(3-i)(3+i)} \right|$$

$$= \left| \frac{-2i}{10} \right| \quad = \sqrt{(0)^2 + (-0.2)^2} \quad = 0.2$$