Calculus Rectangular Complex Number Practice #2

1. $g(x)$ is a quadratic such that $g(4 + 6i) = 0$, write $g(x)$ in expanded polynomial form.

2. Show that $5x^2 + kx - 9$ will never have complex roots.

3. Write $\frac{k + 2i}{2 - i}$ in the form $a + bi$.

4. Find k if $2 + 2i$ is a solution to $x^3 - 8x^2 + 24x + k = 0$.

5. Find c if $(x + 6 + i)$ is a factor of $x^3 + cx^2 + 73x + 111$.

6. If $w = 3 + 2i$ and $v = 4 + k\,i$ and w^2v is entirely real, find k.

7. $4x - \sqrt{3}i$ is a factor of $16x^3 + ax^2 + bx + 12$. What are a and b?

8. Show that $\frac{1 + i + i^2 + i^3 + i^4 + i^5}{1 - i} = i$

Answers: Calculus Rectangular Complex Number Practice #2

1. $g(x)$ is a quadratic such that $g(4 + 6i) = 0$, write $g(x)$ in expanded polynomial form.

   If $g(x) = 0$, then we have a root and also conjugate: $g(x) = (x - 4 - 6i)(x - 4 + 6i)$

   $= x^2 - 4x + 6xi - 4x + 16 - 24i - 6xi + 24i - 36i^2$ $\quad g(x) = x^2 - 8x + 52$

2. Show that $5x^2 + kx - 9$ will never have complex roots.

   $\Delta = k^2 - 4 \times 5 \times {}^-9 = k^2 + 20$ $\quad$ As $k^2 + 20 > 0$ for all k, there are always real roots

3. Write $\frac{k + 2i}{2 - i}$ in the form $a + bi$.

   $\frac{k+2i}{2-i} \times \frac{2+i}{2+i} \quad = \frac{2k + ki + 4i + 2i^2}{4 - i^2} \quad = \frac{2k - 2 + ki + 4i}{5} \quad = \frac{2k-2}{5} + \frac{k+4}{5}\, i$

4. Find k if $2 + 2i$ is a solution to $x^3 - 8x^2 + 24x + k = 0$.

   $(2+ 2i)^3 + 4(2+ 2i)^2 + 24(2+ 2i) + k = 0$ $\quad ({}^-16+ 16i) - 8(8i) + 24(2+ 2i) + k = 0$

   Real parts: ${}^-16 + 48 + k = 0$ $\quad k = {}^-32$

5. Find c if $(x + 6 + i)$ is a factor of $x^3 + cx^2 + 73x + 111$.

   Factor Theorem: $({}^-6 + i)^3 + c({}^-6 + i)^2 + 73({}^-6 + i) + 111 = 0$

   $({}^-198 + 107i) + c(35 - 12i) + 73({}^-6 + i) + 111 = 0$

   ${}^-198 + 35c + {}^-438 + 111 = 0$ $\quad \Rightarrow c = 15$

6. If $w = 3 + 2i$ and $v = 4 + k\,i$ and w^2v is entirely real, find k.

   $w^2v = (3 + 2i)^2(4 + ki) = (5 + 12i)(4 + ki) = 20 + 48i + 5ki + 12ki^2$

   Entirely real, so imaginary part is zero: $48i + 5ki = 0$. $\quad \Rightarrow k = 9.6$

7. $4x - \sqrt{3}i$ is a factor of $16x^3 + ax^2 + bx + 12$. What are a and b?

   $f(\tfrac{1}{4}\sqrt{3}i) = 0$ by Factor Theorem so $16(\tfrac{1}{4}\sqrt{3}i)^3 + a(\tfrac{1}{4}\sqrt{3}i)^2 + b(\tfrac{1}{4}\sqrt{3}i) + 12 = 0$

   $\frac{16 \times 3\sqrt{3}\ i^3}{64} + \frac{a \times 3\ i^2}{16} + b\tfrac{1}{4}\sqrt{3}i + 12 = 0$ Comparing real and imaginary parts

   $\frac{-3a}{16} + 12 = 0$ $\quad \Rightarrow a = 64$ $\quad \frac{-3\sqrt{3}\ i}{4} + \frac{b\sqrt{3}\ i}{4} = 0$ $\quad \Rightarrow b = 3$

8. $\frac{1 + i + i^2 + i^3 + i^4 + i^5}{1 - i} \quad = \frac{1 + i + (-1) + (-i) + (1) + (i)}{1 - i}$

   $= \frac{1+i}{1-i} \quad = \frac{(1+i)}{(1-i)}\frac{(1+i)}{(1+i)} \quad = \frac{2\,i}{2} \quad = i$