Calculus Polar Complex Number Practice #3

1. If the argument of $z = 3 + ki$ is 1, what is k?

2. $w = H \text{ cis } (\frac{8\pi}{5})$ and $v = H \text{ cis } (\frac{5\pi}{6})$. Calculate the exact value of wv in simplest form.

3. $w = n \text{ cis } (\frac{\pi}{5})$ and $v = 2n \text{ cis } (\frac{\pi}{4})$. Calculate the exact value of $\frac{w}{v^2}$.

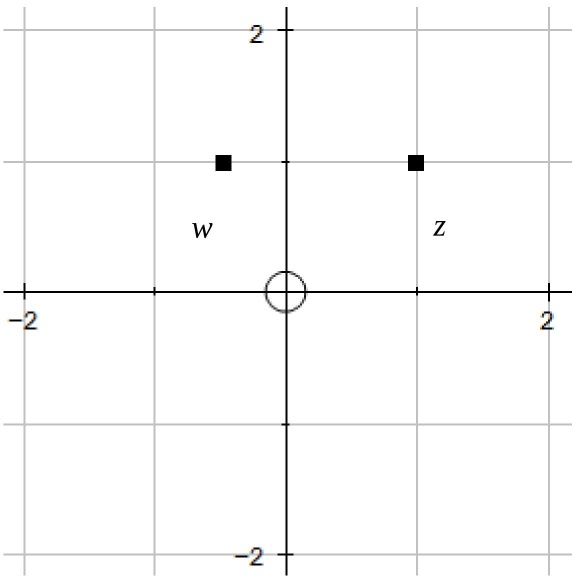
4. What are a and b if $z = a \text{ cis } (\frac{\pi}{6}) = 5 + bi$?

5. Find z such that $z^3 + 8i = 0$

6. Write a general solution to $z^4 + n = 0$ where n is a positive real number.

7. If $u = 8 + ki$ and $v = {}^{-}6 + 3i$, find k if $\arg(u.v) = \pi$.

8. The complex numbers w and z are shown. Plot the point $w + \bar{z}$ on the diagram.



Show your working graphically.

Answers: Calculus Rectangular Complex Number Practice #3

1. If the argument of $z = 3 + ki$ is 1, what is k?

   $\arg z = 1 = \tan^{-1}(\frac{k}{3})$ $\quad\Rightarrow k = \tan(1) \times 3 =$ **4.6722**

2. $w = H \text{ cis } (\frac{8\pi}{5})$ and $v = H \text{ cis } (\frac{5\pi}{6})$. Calculate the exact value of wv in simplest form.

   $w.v = (H \times H) \text{ cis } (\frac{8\pi}{5} + \frac{5\pi}{6}) \quad = H^2 \text{ cis } (\frac{73\pi}{30}) \quad = \boldsymbol{H^2 \text{ cis } (\frac{13\pi}{30})}$

3. $w = n \text{ cis } (\frac{\pi}{5})$ and $v = 2n \text{ cis } (\frac{\pi}{4})$. Calculate the exact value of $\frac{w}{v^2}$.

   $\frac{w}{v^2} = (\frac{n}{(2n)^2}) \text{ cis } (\frac{\pi}{5} - 2 \times \frac{\pi}{4}) \quad = \boldsymbol{\frac{1}{4n} \text{ cis } (\frac{-3\pi}{10})}$ or $\boldsymbol{\frac{17\pi}{10}}$

4. What are a and b if $z = a \text{ cis } (\frac{\pi}{6}) = 5 + bi$?

   $a \cos(\frac{\pi}{6}) + a \sin(\frac{\pi}{6})\, i = 5 + bi$ $\quad$ so $5 = a \cos(\frac{\pi}{6})$ $\quad\Rightarrow \boldsymbol{a = 5.7735}$

   and $b = a \sin(\frac{\pi}{6}) = 5.7735 \sin(\frac{\pi}{6})$ $\quad\Rightarrow \boldsymbol{b = 2.887}$

5. Find z such that $z^3 + i = 0$.

   $\Rightarrow z^3 = {}^{-}8i = 8 \text{ cis } (\frac{3\pi}{2}) = 8 \text{ cis } (\frac{-\pi}{2})$ $\quad z = \sqrt[3]{8} \text{ cis } (\frac{3\pi}{2} \div 3)$ by De Moivre

   $\boldsymbol{z_1 = 2 \text{ cis } (\frac{\pi}{2})} = 2i$ $\quad \boldsymbol{z_2 = 2 \text{ cis } (\frac{7\pi}{6})}$ $\quad \boldsymbol{z_3 = 2 \text{ cis } (\frac{11\pi}{6})} = 2 \text{ cis } (\frac{-\pi}{6})$

6. Write a general solution to $z^4 + n = 0$ where n is a positive real number

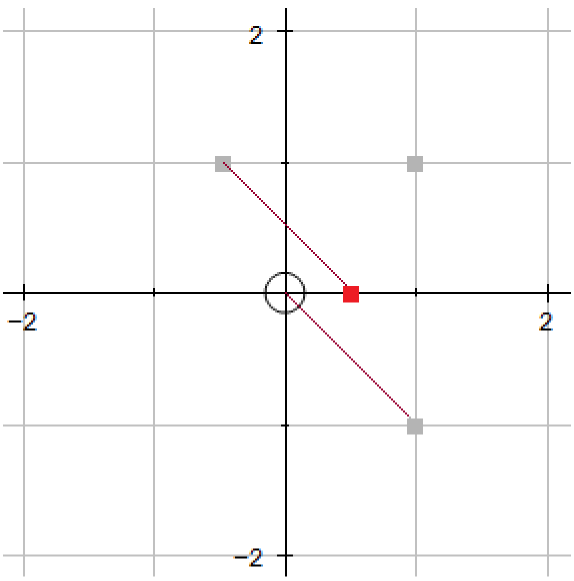
   $z^4 = n \text{ cis } (\pi)$ $\quad\Rightarrow$ by de Moivre: $z = \sqrt[4]{n} \text{ cis } (\pi \div 4)$ is first solution

   Other solutions are at multiples of $\frac{2\pi}{4}$ $\quad\Rightarrow \boldsymbol{z = \sqrt[4]{n} \text{ cis } (\frac{\pi}{4} + \frac{x\pi}{2})}$ $\quad x \in \mathbb{Z}$

7. If $u = 8 + ki$ and $v = {}^{-}6 + 3i$, find k if $\arg(u.v) = \pi$.

   $u.v = (8 + ki)({}^{-}6 + 3i) = ({}^{-}48 - 3k) + (24 - 6k)i$

   $\arg(u.v) = \pi$, the result is a negative real, so $\text{im}(u.v) = 0$ $\quad\Rightarrow 24 - 6k = 0 \Rightarrow \boldsymbol{k = 4}$



8. The complex numbers w and z are shown. Plot the point $w + \bar{z}$ on the diagram.

   $\bar{z}$ is z reflected in real (x) axis

   To add $\bar{z}$ is to go across 1, down 1

   Applying that to w gives red dot.