Calculus Fractions Practice #2

Solve:

1. $\frac{2}{x} + 4 = 7x$

2. $\frac{\frac{2}{7}}{\frac{3}{x}} = 6$

3. $\frac{2x}{x+5} + \frac{3}{x+1} = 8$

4. $\frac{\frac{5}{x+2}}{\frac{x-1}{2}} = {}^{-}2$

Write for y in terms of x, as a simplified single fraction:

5. $\frac{x}{x+y} = 4$

6. $\frac{1}{\frac{3}{x}+\frac{2}{y}} = 5$

7. $\frac{2}{x-1} + \frac{x}{y} = 2$

8. $\frac{1}{xy} + \frac{1}{y} = 5$

Rearrange and simplify to give these to their simplest possible two line fractions (note, they become very simple fractions in most cases):

9. $\frac{\frac{1}{x^2}+\frac{2}{x}}{\frac{1}{x}+\frac{2}{x^2}}$

10. $\frac{a}{a-b} + \frac{b}{b-a}$

Proofs:

11. Show that: $\frac{\frac{1}{b}-\frac{1}{a}}{\frac{1}{b}+\frac{1}{a}} = \frac{a-b}{a+b}$

12. Show that: $\frac{uy - uz + xy - xz}{uy - uz + yz - y^2} = \frac{u+x}{u-y}$

Answers: Calculus Fractions Practice #2

There are multiple ways of answering these questions, but usually one is much easier than the others.

1. $\frac{2}{x} + 4 = 5x$ $\quad 2 + 4x = 5x^2$ $\quad 0 = 5x^2 - 4x - 2$ $\quad \mathbf{x = 1.148 \text{ or } {}^{-}0.348}$

2. $\frac{\frac{2}{7}}{\frac{3}{x}} = 6$ $\quad \frac{2}{7} = 6 \times \frac{3}{x}$ $\quad \frac{7}{2} = \frac{x}{18}$ $\quad \mathbf{x = 63}$

3. $\frac{2x}{x+5} + \frac{3}{x+1} = 8$ $\quad \frac{2x(x+1)+3(x+5)}{(x+5)(x+1)} = 8$ $\quad 2x^2 + 5x + 15 = 8x^2 + 48x + 40$ $\quad \mathbf{x = {}^{-}6.528 \text{ or } {}^{-}0.638}$

4. $\frac{\frac{5}{2-x}}{\frac{x-1}{2}} = {}^{-}2$ $\quad \frac{5}{2-x} = \frac{-2x+2}{2}$ $\quad 10 = 2x^2 - 6x + 4$ $\quad \mathbf{x = 3.791 \text{ or } {}^{-}0.791}$

5. $\frac{x}{x+y} = 4$ $\quad x = 4x + 4y$ $\quad {}^{-}3x = 4y$ $\quad \mathbf{y = \frac{-3x}{4}}$

6. $\frac{1}{\frac{3}{x}+\frac{2}{y}} = 5$ $\quad \frac{3}{x} + \frac{2}{y} = \frac{1}{5}$ $\quad \frac{2}{y} = \frac{x-15}{5x}$ $\quad \mathbf{y = \frac{10x}{x-15}}$

7. $\frac{2}{x-1} + \frac{x}{y} = 2$ $\quad \frac{x}{y} = \frac{2x-2}{x-1} - \frac{2}{x-1}$ $\quad \frac{y}{x} = \frac{x-1}{2x-4}$ $\quad \mathbf{y = \frac{x^2 - x}{2x-4}}$

8. $\frac{1}{xy} + \frac{1}{y} = 5$ $\quad \frac{1}{xy} = \frac{5y-1}{y}$ $\quad \frac{1}{x} = 5y - 1$ $\quad \mathbf{y = \frac{x+1}{5x}}$

9. $\frac{\frac{1}{x^2}+\frac{2}{x}}{\frac{1}{x}+\frac{2}{x^2}}$ $\quad$ (multiply by $\frac{x^2}{x^2}$) $\quad = \frac{\frac{x^2}{x^2}+\frac{2x^2}{x}}{\frac{x^2}{x}+\frac{2x^2}{x^2}}$ $\quad = \mathbf{\frac{1+2x}{x+2}}$

or give common denominators $\frac{\frac{1}{x^2}+\frac{2x}{x^2}}{\frac{x}{x^2}+\frac{2}{x^2}}$ and now divide out common x^2

10. $\frac{a}{a-b} + \frac{b}{b-a}$ $\quad = \frac{a(b-a)+b(a-b)}{(a-b)(b-a)}$ $\quad = \frac{ab-a^2+ab-b^2}{ab-a^2-b^2+ab}$ $\quad = \mathbf{1}$

11. $\frac{\frac{1}{b}-\frac{1}{a}}{\frac{1}{b}+\frac{1}{a}}$ $\quad = \frac{\frac{a}{ab}-\frac{b}{ab}}{\frac{a}{ab}+\frac{b}{ab}}$ $\quad = \frac{a-b}{a+b}$

or multiply top and bottom by $\frac{ab}{ab}$ and rearranged bottom line

12. $\frac{uy - uz + xy - xz}{uy - uz + yz - y^2}$ $\quad = \frac{u(y-z)+x(y-z)}{u(y-z)+y(z-y)}$ $\quad$ recalling that $(z-y) = -(y-z)$

$= \frac{u(y-z)+x(y-z)}{u(y-z)-y(y-z)}$ $\quad = \frac{(u+x)(y-z)}{(u-y)(y-z)}$ $\quad = \frac{u+x}{u-y}$